Episode 6: Letting Algorithms Emerge Naturally

Pre-Viewing Reflection

Mathematical algorithms are formidable interventions of the human mind. They are testimonies to mathematical insight, ingenuity, and efficiency. Yet mathematics educators, such as Constance Kamii and Ann Dominick, warn us of their potential harm, as "they encourage children to give up their own thinking." But the algorithms themselves are not harmful—rather, the potential problems lie in when, how, and in what context we introduce them.

If there is one area of school mathematics in dire need of a shift "from rote to reason," it's fractional computation. Traditional Instruction of fraction algorithms has essentially been based on rules:

- To add or subtract two fractions, first find common denominators.
- To multiply two fractions, just multiply the numerators and multiply the denominators.
- To divide two fractions, invert the second one, then multiply both fractions.

Lost in this jungle of rules, students often relinquish their own sense-making and surrender to memorization. This contributes to the belief that mathematics is more about memorizing procedures than reasoning with or about powerful ideas.

Reflect on the following questions before watching how Dr. Monica Neagoy helps a group of fifth grade students make sense of adding and multiplying fractions.

1. What prerequisite notions must be in place before formally introducing fraction operations?

___

___

___

2. Do you build on students' knowledge of whole number operations when introducing fraction operations?

___

___

___

3. Do you make explicit the similarities and differences between operating with whole numbers and operating with fractions (both conceptual and procedural)?

4. More precisely, do you review the different models or meanings of each operation as well as contexts and situations that can be represented by each one of them? For example, one model or meaning of "a + b" (addition) is to "join, combine, or put together two quantities or collections to make a third (the sum);" and one model or meaning of "a ÷ b" (division) is to "find how many equal parts/groups of size b can be made from a whole/set of size a."

5. Do you ever give students the opportunity to first invent a story or a situation that gives meaning to—and can be modeled by—an expression such as "$\frac{1}{2} + \frac{1}{4}$," "$6 \times \frac{2}{8}$," or "$3 \div \frac{1}{2}$"? If so, do students focus more on meaning and understanding or on process and procedure when sharing their thinking?

6. Do you use a variety of concrete, tactile, or visual models to illustrate the different operations (pattern blocks, tangrams, connecting cubes, Cuisenaire rods, geoboards, bar models or tape diagrams, number lines, arrays and rectangular grids, and so forth)? Do you carefully plan the sequencing of problems introduced and models used?

7. Do you find that many students practically equate understanding multiplication (or another operation) of fractions with knowing the algorithm, applying it correctly, and producing the right answer? How might you increase both the breadth and the depth of their understanding?

8. Have you often witnessed students confused by the rules, or by the steps within the rules, when working with fraction algorithms? What are their most frequent types of misconceptions?

Making Sense of Operations Before Introducing Algorithms

As you watch the video, record the strategies Dr. Monica uses to help students understand the meanings and procedures of adding and multiplying with fractions. Jot down what struck you most in terms of the teacher's actions and discourse and in terms of students' responses and thinking. Do you think some students made progress in their concept development after that lesson (of which about one-third was captured on video)? On the following chart, note any general or specific observation not necessarily related to fractions. For instance: how Dr. Monica addressed two students' erroneous responses to whether or not the fractions ½ and ¼ needed to refer to the same whole (when adding them).

Task	Teachers' Actions and Discourse (task / strategy / model / words / etc.)	Students' Responses / Thinking (indications of concept development)
Addition of fractions		
Multiplication of a whole number by a fraction		
Picnic problem (involving two fractions)		
Other Observations and Reflections		

Post-Viewing Reflection

1. Dr. Monica began the lesson by projecting an addition expression on the board and asking students to create a story that would give meaning to the expression. What do you think of such an approach? Is it an approach you use in teaching mathematics? If not, might you consider it? If so, why?

2. How do you balance meaning making with learning efficient rules when working with students on fraction operations (or on other mathematical topics)? Do you find that one is more dominant than the other in the daily classroom "math talk"?

____

____

____

3. What strategies or approaches did you see in the video that seemed effective in building understanding? Why do you think they were effective?

____

____

____

4. Think about how you might guide your students in inventing a story or situation to give meaning to expressions such as "$3 \div \frac{1}{2}$", "$\frac{1}{2} \div 3$", or "$\frac{1}{2} \times \frac{2}{3}$". Can you see how the division algorithm, for example, might emerge seamlessly without your "telling it"?

____

____

____

Suggested Reading

Chapter 6 of Dr. Monica's book *Unpacking Fractions* (ASCD, 2017) shares nontraditional ways of teaching the meanings and operations with fractions and offers reasonable tasks that all students can engage in and that allow the rules (algorithms) to emerge naturally. As in all chapters, she first relates a fraction lesson on the subject (with student and teacher discourse), then identifies students' common misconceptions, explains the underlying mathematics in depth, and finally offers challenging questions to help students tackle their misconceptions.

Chapter 6 is rich with ideas and practical tips that every teacher will appreciate and apply. In particular, you will find the answers to all questions posed above, and more. If you don't have time to read the entire chapter, you may be interested in the following section: Unpacking the Mathematical Thinking. Its four goals are to:

- Offer four problem situations that are accessible to all children;
- Share students' invented strategies for solving these problems, divided into common types of problem-solving tactics;
- Review operation meanings, properties, and relationships; and
- Highlight two important changes in ways of thinking that are brought on by rational numbers.