CURRENT

THE revolution in the teaching of elementary and secondary school mathematics, commonly known as the "new math," is now approaching the quarter century mark—one of its earliest efforts being the work of Max Beberman at the University of Illinois beginning in 1951. Like most revolutions, its causes are multiple and somewhat obscure.

G. Baley Price (1961) cited three causes: the tremendous advances made in mathematical research in the 20th century; the automation revolution which made it theoretically possible to construct machines with enormous capabilities, and hence the need to create such machines; and the development of automatic digital computing machines making computations more quickly and efficiently than any group of humans could do—such as the computations guiding manned vehicles to the moon's surface.

Another more visible and affect-laden cause often cited is Russia's first effort in space, Sputnik in 1957, with its implications for our need to maintain superiority in the cold war of the times.

Three Thrusts of the New Math

The new math revolution can be viewed as three interrelated thrusts. The first objective of the scholars leading the revolution was that of developing alternative textbook programs that were more mathematically correct than those in use in the first half of the century. The structure of the discipline, not the expressed needs of the learner and not the adult needs of a career-oriented society, was to be the dominating source of the curriculum.

Hopefully then, the teacher would teach and the child would learn the mathematical rationale for the operations rather than simply the mechanical manipulation of the symbols. While learning to multiply factors of the form $3 \times 14 = n$, the child would also learn the distributive property of the real number system.

A concomitant of the new emphasis on the structure of the discipline was the stress on the cleaning and polishing of the mathematical language. Teachers were discouraged from using such expressions as "borrowing," "reduce to lowest terms," "invert the divisor and multiply," "cancel," and from referring to numerals as numbers.

Leading the way in the preparation of the experimental programs were the University of Illinois group and the School Mathematics Study Group (SMSG), Edward G. Begle, Director. These programs were the products of teams consisting of mathematics educators, mathematicians, cognitive and developmental psychologists, classroom teachers, and other school personnel.

A number of "experimental" programs were developed for use on a narrow range of grade levels; still others consisted of a few interesting topics in mathematics. The planning and writing teams for these projects varied from large, well-balanced teams to essentially idiosyncratic efforts, the latter being what Harvard dean Ted Sizer (1965) referred to as "gee-whiz" programs.

To keep the educational community informed on the characteristics and progress of these programs, the National Council of Teachers of Mathematics published the titles *An Analysis of New Mathematics Programs* (1963), and later *The Continuing Revolution in Mathematics* (1968).

STATUS OF THE NEW MATH

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The intent of the directors of the larger, more substantive projects was to create, try out, and evaluate programs that would offer the commercial publishers alternatives to their present programs. Later, in several instances, authors of the alternative projects became authors of newer commercially published programs which were heavily influenced by the projects.

The objectives of the scholars, to create more discipline-oriented programs, having been accomplished, most of the federal and foundational funding of the projects was discontinued, and rightly so.

By and large, the new math programs were aimed at the above average student. During the past five years the textbook publishers, recognizing the needs of the slow children and youth, have published more appropriate materials.

The second thrust of the revolution was that of strengthening the mathematical backgrounds of preservice and in-service teachers to be able to use effectively the newer programs. The Committee on the Undergraduate Program in Mathematics (CUPM), a committee of the Mathematical Association of America, funded by the National Science Foundation, took the lead in establishing recommendations for course content. A substantial improvement was effected in some preservice teacher education programs, a modest improvement in many, and little or no change in still others.

The federal, local, and state governments as well as private foundations carried out massively funded efforts to strengthen the mathematical backgrounds of teachers-in-service.

This second thrust of the revolution was much more difficult to carry out than the improvement of the textbook materials, and was perhaps less successful.

The third thrust was that of improving the teacher's ability, including both cognitive and noncognitive variables, with the anticipated result of measurably increased learning on the part of children and youth. This was by far the most complex and perhaps least successful of the three thrusts of the new math.

A multitude of cognitive-developmental psychologists, psychologically-oriented mathematics educators, and school personnel engaged in this large uncoordinated effort. Whereas both of the other thrusts, improving the textbook programs and strengthening the teacher's background, could be carried

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out as somewhat independent efforts, the improvement of the teacher's ability to teach the new math had to be viewed as the interaction of many variables, cognitive and noncognitive: the learner—his native ability, his acquired cognitive abilities, his attitudes and aptitudes, his readiness for learning (psychological, sociological, and cognitive), his learning style, and others; the teacher—his humaneness or lack of it, his knowledge of appropriate mathematics, his skill in diagnostic teaching, his skill in establishing a good fit between the learner's talent and the task, and others; the curriculum—the breadth and appropriateness of its objectives, its content, its evaluation both formative and summative, its flexibility, and others.

Several people were "pre-revolution" leaders in this thrust; above all, Jean Piaget, Swiss genetic-epistemologist and cognitive-developmental psychologist, who presented to the teaching profession over almost a half century a new theory of intelligence—intelligence as logical thinking. Because he used mathematical concepts as the vehicle for his investigations, his research findings were and are particularly attractive to the community of mathematics educators.

William A. Brownell, a superb cognitive-developmental psychologist, beginning in the late 1920's carried out a monumental series of studies of complex classroom learning of mathematics. These studies of meaningful (conceptual, new math) learning versus mechanical (associational, "old" math) learning served as guideposts for the nouveau-venu cognitive psychologists of the new math.

Most visible among the professional books that provided the psychological rationale for the new math was *The Process of Education* (1960). In this small influential volume, Jerome S. Bruner summarized the thoughts of 34 scientists, scholars, and educators who met at Woods Hole, Cape Cod, to discuss how science education in the schools might be improved. The second chapter, "The Importance of Structure," presented a compelling argument for making the structure of the discipline the primary source of the mathematics curriculum.

In a companion volume, *On Knowing: Essays for the Left Hand* (1962), Bruner presented a rationale for the new method for the new math—discovery. Here is stressed the idea that knowing is a process, not a product.

The input of effort and money into the realization of this third thrust of the new math revolution was substantial. There is, however, little hard evidence that the implementation of the objectives of the scholars in the improvements of the teacher's teaching and the amount of the child's learning was as successful as the first thrust, the improvement of the mathematics textbook programs.

Evaluation of the New Math

Only a small fraction of the effort and money that went into the development of the alternative textbook programs, the first thrust of the new math revolution, went into the evaluation of those materials. This small investment yielded a corpus of evaluative research with outcomes so complex that they can only be lightly touched upon here. The reader is referred to a general summary by Edward G. Begle and James W. Wilson (1970).

Perhaps the most educationally significant study was the National Longitudinal Study of Mathematical Abilities (NLSMA). This study was undertaken "to obtain quantitative information which would be useful in further curriculum development and also would be useful to school administrators in making decisions about curriculum problems." A large number of fourth and seventh graders were tested each fall and spring for five years. Another large group of tenth graders was tested for three years, with a post-high school follow-up by means of a questionnaire. Tests, constructed by a panel of mathematics educators, contained subtests of types of learning from computational skills to the higher mental processes of comprehension, application, and analysis.

Begle and Wilson offered several general observations on the results derived from the NLSMA study:

In general, the results for the SMSG textbooks were favorable. But not all modern textbooks (of those included in the study) produced the kind of results that were expected for them. Some of them in fact did rather poorly on *all* levels—from computation to analysis. Those textbooks which did not do very well were for the most part considerably more formal and more rigorous than the SMSG textbooks. In the upper grades, more attention was paid to axiomatics and strict deduction than is paid by SMSG, and less, apparently, to understanding the basic concepts.

No large-scale studies yielding generalizable data have been carried out that would assist us in determining how effective the second and third thrusts of the new math were in improving the quantity or quality of children's and youths' learning.

Future of the New Math

While a knowledge of the recent history of mathematics education cannot be used to determine the future of the interdiscipline, a lack of that knowledge may occasion us to repeat one or more of the failures of the past.

Admittedly, it is risky to engage in futurism, but a few guesstimates will be offered. The first thrust is largely ended. It is unlikely that any major large-scale new revolution of the 1960's variety will be mounted in the near future to further formalize the textbook programs. On the contrary, some now fear that the influence of the mathematician on the new math was too great. A retrenchment may be indicated. Van Engen (1972), mathematician and mathematics educator, expressed his concern:

> As much progress should be made in problems of pedagogy and curriculum in the next decade as has been made in mathematical quality in the past decade. Most certainly, there is reason to question the degree of formalism that is creeping into the elementary school. . . . Furthermore, the rapid pace of the more usual programs is questionable.

A number of attempts have been made to individualize the teaching of the new mathematics through *modified* text-type materials. Suydam and Weaver (1969) summarized the research on these programs:

> All in all, there is little substantial evidence to date indicating that programs of individualized mathematics instruction will lead to higher levels of pupil achievement when compared with non-individualized programs.

Of these programs, IPI (Individually Prescribed Instruction) is perhaps the most widely publicized and advertised. In the IPI program, not only do children learn no more mathematics than children in conventional programs, but the child's self-concept *lowers* more each year that he is exposed to it (Myers, 1972).

The textbook will continue to be the teacher's main tool for ensuring the systematic, sequential, and structured teaching of mathematics and for consolidating the learning. John I. Goodlad (1965) said,

> School principals have listed textbooks as the resource most useful and influential in the teaching program. Some publishers are now producing *several levels* of materials dealing essentially with the same concepts . . . from which to choose those most pertinent to the learning needs of individuals and groups.

Perhaps most significant to the future of the new math will be the impact of the new humanism. Two recent yearbooks of ASCD, although not concerned with mathematics alone, indicate a shift of the pendulum away from the extreme formalism of the new math of the 1950's-1960's: *A New Look at Progressive Education* (Squire, 1972), and *To Nurture Humaneness* (Scobey and Graham, 1970).

Nor is the rationale for moving away from the structure of the discipline as the organizing principle of curriculum entirely a philosophical one. Rohwer (1971) has summarized and interpreted the research on the cognitive development of children and youth. Central among his hypotheses are: ". . . the longer we delay formal instruction, up to certain limits, the greater the period of plasticity and the higher the ultimate level of achievement." And the converse: ". . . if formal schooling that seeks to accelerate intellectual development prior to the adolescence transition results in less plasticity and a lower final level of intellectual capacity, then it should surely be discarded."

The move away from *extreme* formalism should prove to be salutary. Hopefully, the pendulum will come to rest at some reasonable midpoint in the arc lest it continue on and we witness again the excesses of a *misunderstood* progressivism of the 1920's to 1940's.

We can have the best of both worlds if we remain informed, courageous, and vigilant.

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