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Mathematics K-14

Reform in mathematics is fundamental.

EVERY country in the world, faced with the transition to a modern economy, finds that reform of the mathematics curriculum is fundamental for this transition. The problems vary from country to country, depending on where they are now. But hardheaded economists, who must plan for national prosperity, find these problems crucial. That is why the OEEC organized an international conference in Paris, November 1959, on "New Thinking in School Mathematics" (14). Every schoolman should study this report.

1. Why teach mathematics?

Our basic problem is that we live in the most rapidly changing society in history. We can no longer predict what an adult should know, and then teach this to children. To prepare for a changing world, a child must learn to deal with problems for which he has not been specifically instructed. He must learn how to learn, and that learning is important and rewarding. He must love learning for its own sake, since no one can predict the payoff to him for anything he learns.

Thus the emphasis in the curriculum must be on the tools and motivation for lifelong learning. A main function of the mathematics curriculum is to teach one of the two principal ways of acquiring knowledge at firsthand—the method of logical reasoning. Skills in computation and factual knowledge are important as means to this end.

The main secondhand method of acquiring knowledge is through the language arts—learning what others have found out. Mathematics is a language as well as a science. The literature in most fields is now written in mathematical language. Unless a person is literate mathematically, he cannot read about current developments in his field of interest.

With the advent of automation, routine work is disappearing in business and industry. Mathematical education is essential for most categories of skilled and professional manpower.

In our country all citizens have a voice in social decisions. Now about 80 percent of the bills before Congress involve science and mathematics. A person cannot even understand the issues unless he is scientifically and mathematically lit-

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erate. Whether he uses mathematics vocationally or not, he must have the background to make wisely the decisions he must make as a citizen (1, 5, 9, 11, 12, 22).

2. How is mathematics changing?

Our present mathematics curriculum is adapted to the America of 1910, when 90 percent of the population were engaged in unskilled labor and farming, a large proportion of children left school before the 9th grade, the apprenticeship system was breaking down, and the assembly line was decreasing the demand for skilled labor. Our geometry program is a watered down version of Euclid, and our algebra program is about 300-400 years old. Virtually the only applications mentioned in school are to shopping and surveying. In calculus the applications are to the engineering of 50 years ago.

Almost all of the new developments in the mathematics curriculum are very conservative, and aim primarily at improving the teaching of the same content in the light of the development of mathematics from 300 B.C. to about 1905. The general structure of the curriculum is not disturbed very much—arithmetic up to grade 8, algebra in grades 9 and 11, geometry in grade 10. Most of the changes now proposed are not as radical as many schoolmen believe.

Although minor differences loom large to partisans of the various projects, their common features are much more important. They all aim at—

1. Giving the student a coherent structure which will make it easier to learn new things and remember the old.

2. Emphasis on reasoning, beginning rather informally in elementary and junior high school, and leading to formal proof in algebra and geometry.

3. Presenting mathematics as a creative art, rather than as a finished product, by giving students experience in discovery.

4. Unifying subjects such as arithmetic, algebra, and geometry, which are traditionally taught separately.

5. Clarifying the language of school mathematics, which is sloppy and confused in the conventional curriculum.

The various new proposals differ in their relative emphasis on logic and intuition, explicit instruction in language, and balance between abstraction and concreteness. Geometry will be in the curriculum from kindergarten up. Some algebra may be introduced as early as second grade, in connection with subtraction and division:

$5-2$ is the solution of the equation $2+x=5$,

$6\div 2$ is the solution of the equation $2\cdot x=6$.

The new geometry courses include much more algebra than the conventional courses. High school algebra is moved back from college to high school, where it was 25 years ago and where it belongs.

For students capable of college, the aim is that calculus, with analytic geometry, be the standard beginning course in college. Actually the top 10-15 percent of the 16-year-olds can learn calculus, but we shall not be prepared to teach it to them for some years to come.

The most important new content now proposed is the introduction of probability and statistics in the SMSG courses for grades 7–8 and as one alternative for 12th grade (18, 7).

We have analyzed in this journal (17) in some detail the trends in the elementary curriculum. W. H. Meyer (13) has written an excellent summary of the common elements of the new secondary curricula. A convenient reference on projects in mathematical education has been published by Scott, Foresman and Co. (21). Recommendations for the curriculum in grades 13–14 will soon be published by the Committee on the Undergraduate Program in Mathematics (CUPM), directed by Dr. R. Wisner, University of Michigan Oakland, Rochester, Michigan. We recommend the bulletin (19) for information on current developments.

3. What to do now?

The most thoroughly studied part of the curriculum is that for college-capable students in grades 9–12. Every mathematics teacher and curriculum supervisor must be familiar with the reports of the College Entrance Examination Board (6), and with the major projects such as those of SMSG (18), Illinois, and Ball State (26, 3, 4). The best way to learn the new developments is to teach one of the new courses. Our evidence so far is that these courses do not do a student harm, probably do him good, and certainly teach the teacher a great deal. Whatever a school's ultimate choices may be, it should certainly begin trying the new courses now.

The Illinois and the Ball State materials make more of a demand on the teacher's background than do the SMSG courses. Our evidence in Minnesota is that ordinary teachers can teach SMSG effectively with some consultant service (one meeting per month) during the first year of experience. An ordinary teacher should probably attend a summer institute before trying one of the other two. Some recent commercial textbooks along these lines are intermediate in their demands on the teacher.

We find that the teacher's attitude is a more important factor than his formal preparation in his effectiveness with the new courses. If he is flexible, willing to try new things, and exercises critical judgment, he is likely to do well. If he is rigid, resistant to change, or uncritically for or against, he is likely to do poorly. Where the teacher has used the new materials because of pressure from administrators or parents, results have not been as favorable.

The teacher has a duty to be informed of developments in his field. The administrator has the right and the duty to insist on this since he is responsible for the professional competence of his staff. If he cannot require membership in the National Council of Teachers of Mathematics, he may offer incentives such as school sharing in the payment of dues and school subscription to the *Mathematics Teacher*. He should certainly see to it that teachers take full advantage of opportunities for in-service education, such as those provided by the National Science Foundation. If possible, he should have at least two teachers in his school teaching the new courses. He should arrange for staff seminars, as part of the normal work load, in which teachers can teach each other the new developments. He may have to relieve his staff of some clerical or other duties so that they will have time for this activity.

We recommend that at least two teachers begin a new program simultaneously so that they can share experiences and discuss problems with each other. Also, if one is absent, the other can take over, while substitutes can be used in conventional courses.

It is probably best to begin at a transitional grade—grade 9 in an 8-4 school system, and grade 10 in a 6-3-3 school system. SMSG can be started at any grade. The Illinois project has a much more closely integrated program for grades 9–12. The Ball State materials for grades 9–11 are not quite so interdependent, but probably should be started at grade 9. Some recent textbooks are quite suitable for introduction at a particular grade level with assumption of a conventional curriculum up to that point.

In grades 7–8 SMSG, UMMaP (27), and Ball State (2) are much superior to the conventional courses and are suitable for all students. There are also now available several similar commercial textbooks. Teachers and administrators tend to regard these programs as primarily for high ability pupils. Our experience is that low ability pupils gain even more, in comparison with their achievement in conventional courses, than do high ability pupils. We have had inadequate experience in the use of the SMSG supplementary junior high school units for superior students. The SMSG materials for the non-college bound in grades 7–10 are especially written for students with low reading ability.

The UMMaP program includes a formal introduction to algebra in 8th grade. SMSG has some algebra in grade 8, but leaves the formal instruction in algebra to grade 9. The Ball State 8th grade text overlaps considerably with the SMSG courses for grades 7–8. SMSG has more material on the applications of mathematics than any of the other courses.

Since our previous article (17) appeared, the Greater Cleveland project (10) and the Page project (25) have expanded their activities considerably. The former has produced a large amount of material for grades K-6. My project on grades K-3 is under way (16), and should have materials ready for larger scale use by 1963. The SMSG materials in grades 4–6 have received extensive tryout and revision. Several commercial publishers have followed the lead of Scott, Foresman, and Co. with forward looking elementary series. Suppes (23, 24) and Davis (8) have produced more materials at various levels.

Elementary teachers should have a consultant from a nearby high school or college when they begin one of these programs. Supervisors and principals should be encouraged to attend summer institutes on elementary mathematics.

Since the curriculum is in a state of flux, a school should move toward making an annual reconsideration of texts possible. If the normal life of a text in a system is 5 years, books should be offered to the students for sale at the end of the school year, after grades have been given, at 20 percent off for each year of use. Students should be encouraged to build up their own mathematical libraries. Schools must try to avoid building up inventories of obsolete texts.

It is especially important that the new materials be used in preservice education, in methods courses, student teaching, and laboratory schools. Otherwise we are forced into a permanent program of remedial in-service education.

Films for teacher education have been produced by Beberman, at Illinois, and myself. Davis has produced tapes valuable for this purpose.

4. Where do we go from here?

We must emphasize that most existing projects are geared to what existing teachers can teach, rather than what youngsters can learn. The work of Page, Davis, Suppes, and myself indicates that children can learn much more than most educators have dreamed. We shall not be able to make more fundamental improvements until we can educate teachers properly. The joint recommendations of CUPM-NASDTEC (28) on teacher education in mathematics must be implemented for further progress.

We mention here some directions in which we can expect the curriculum to change further:

1. An alternative curriculum in grades 9–11 with a more concrete approach and with a greater emphasis on the applications of mathematics to the natural and social sciences;

2. A vectorial approach to geometry, with thorough integration of algebra and geometry, and an intuitive approach to vectors beginning in grade 7;

3. Formal instruction in algebra beginning in grade 7, building on the new elementary programs;

4. An integrated mathematics and science curriculum in grades K-9 (see 20);

5. A very concrete senior high school mathematics and science program with emphasis on new needs in industrial arts, business, agriculture, home economics;

6. A cultural course along the lines of Kline and Rademacher-Toeplitz (11, 12, 15), with adequate exercises for students, but making few demands on technical mastery;

7. College courses building on the new high school programs, with more attention to modern applications to the natural and social sciences.

Schoolmen must expect, like the rest of us, to live in an atmosphere of constant change, which makes demands on them, too, for lifelong learning. They cannot hope to attend a summer institute or introduce a new program, and then be set for another 20 years. Schoolmen will have to become scholars. School programs and facilities must be planned with adequate provision for scholarly activity by the staff. School boards and citizens will have to be educated to expect this and to pay for it.

The learning activities of teachers and administrators must serve as models for the students and the community. For our survival in a changing world we need schools run by people who "gladly learn and gladly teach."

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officers, members of the Executive Committee and Board of Directors should continue to be clearly identified in leadership roles in various aspects of the conference program in operation.

4. Encouragement needs to be given to specialists, supervisors and consultants in the various curriculum areas to participate and feel at home in ASCD. Would it be helpful to encourage luncheon groups for NSTA; NCTE; NCSS; NCTM; music, art, etc.?

5. More creative use needs to be made of the new tools of communication throughout the conference. What about closed circuit TV? Why not enliven presentations and rooms by using new machines for displaying charts, etc.? Could we use some prerecorded tapes?

And so another year's work of the Conference Planning Committee comes to a close. Many ideas have been brought forth, some will find expression in action in the program at Las Vegas. Will it be a stimulating conference? The prognosis is good. Whether it is or not will depend not only on the planning that has gone before, but upon the quality of participation of *each* and *every* person attending the conference.

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