Mastering the basic facts for addition, subtraction, multiplication, and division is an essential goal for all students. Most educators also agree that success at higher levels of math hinges on this fundamental skill. But what’s the best way to get there? Are flash cards, drills, and timed tests the answer? If so, then why do students go into the upper elementary grades (and beyond) still counting on their fingers or experiencing math anxiety? What does research say about teaching basic math facts so they will stick?

In Math Fact Fluency, experts Jennifer Bay-Williams and Gina Kling provide the answers to these questions—and so much more. This book offers everything a teacher needs to teach, assess, and communicate with parents about basic math fact instruction, including

- The five fundamentals of fact fluency, which provide a research-based framework for effective instruction in the basic facts.
- Strategies students can use to find facts that are not yet committed to memory.
- More than 40 easy-to-make, easy-to-use games that provide engaging fact practice.
- More than 20 assessment tools that provide useful data on fact fluency and mastery.
- Suggestions and strategies for collaborating with families to help their children master the basic math facts.

Math Fact Fluency is an indispensable guide for any educator who needs to teach basic facts. This approach to facts instruction, grounded in years of research, will transform students’ learning of basic facts and help them become more confident, adept, and successful at math.
MATH FACT FLUENCY

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Basic facts truly are the foundation on which all mathematical computation is based (larger numbers, rational numbers, operations with variables, and so on). However, too many students leave elementary school lacking fluency with the basic facts. Clearly, the historical (and still prominent) approach to teaching basic facts has been ineffective. This is because there are two fundamental flaws in this type of instruction: (1) a lack of attention to strategies, falsely assuming students can go straight from counting (or skip counting) to just knowing the facts; and (2) a lack of effective assessment, falsely assuming that timed tests can provide meaningful data on student mastery of basic facts. Despite the ineffective and potentially damaging effect of these teaching and assessment approaches, basic fact instruction has changed very little over the years. It is time for a change!

This book is structured around five fundamentals for transforming basic fact instruction. These fundamentals are firmly grounded in research and collectively outline a plan for students that results in lasting learning of the facts, without damaging side effects. By exploring the meaning of numbers and operations, describing their thinking, making sense of the strategies of their peers, and engaging in meaningful practice, students eventually reach mastery of the basic facts, all the while becoming empowered to think and act like mathematicians. Chapter 1 provides a brief, clear description of the five fundamentals. Chapters 2 and 3 describe the learning progressions for addition and subtraction facts, including a plethora of strategies, activities, and games, while Chapters 4 and 5 do the same for multiplication and division. The 40+ games in this book (listed at the end of this preface) are fun, but that is not why we included them. Games provide the opportunity to talk about strategies, practice a newly learned strategy, and become more efficient at using strategies until
automaticity with the basic facts has been attained. Did you notice how many times the word strategy was used in the last sentence? It is that important! This word is often used in many different ways in the classroom, but in this book we only use this term to refer to thinking strategies. As you will learn, these thinking strategies are the key to helping students developing lasting fact mastery.

The activities and games featured in this book also serve a secondary purpose: assessment. While students are busy playing games and talking about their thinking, you have an opportunity to employ assessment techniques that will provide far better data than a timed quiz, all the while avoiding the negative impacts of such assessments. Chapters 6 and 7 provide ideas for observation tools, easy interviews, formal interviews, journal prompts, and ways to monitor student progress toward mastery of the foundational fact sets (Chapter 6) and derived fact sets (Chapter 7).

One particular challenge with the basic facts is understanding the myriad of terms: fluency, automaticity, rote memorization, knowing from memory, mental strategies, and mastery. In this book, we focus on the accepted, research-based definition of fluency, which delineates four components: “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately” (National Research Council, 2001, p. 116). This definition expands the focus of mastering facts to include strategies and flexibility, as compared to a focus solely on accuracy and efficiency. Many people, including parents, principals, teachers, politicians, and students, think of fluency as simply being able to say a fact quickly (automaticity). Educating everyone about this more comprehensive notion of fluency is absolutely essential to helping every child learn basic math facts for life and feel confident and competent about their ability to do math. Chapter 8 focuses on ways to engage families and other stakeholders in understanding the importance of fluency, as well as how they can help their own children become fluent with the basic facts.

Our own advocacy for change, through countless presentations and several journal articles, has resulted in many schools adopting different approaches to teaching basic facts, implementing ideas that are now in this book. These instructional programs and assessment tools have resulted in dramatic change in some schools: more students achieving mastery through games and other activities (as opposed to drill or rote memorization), students becoming more excited about mathematics and confident in their abilities, and teachers feeling that they are doing a better job at preparing their students with the basics. For example, a 2nd grade teacher recently emailed Jennifer this message at the end of the year: “I was a very math-averse teacher who had no idea how to teach students concretely about math. That changed this year.
with our focus on explaining reasoning strategies and showing their thinking using a variety of visual models. One of the best compliments I got from a student this year was “I love math.” Teachers at these sites have often asked us, “Will you write a book?” As we pondered undertaking such a task for several years, we realized that a worthwhile book must not only provide explanations but must also be bursting with activities, games, and tools that could be lifted right out of the book and put to use. That is what we have written. We recognize we are asking for fundamental changes to teaching basic facts. We encourage you to reflect on the way in which our five fundamentals align with your school’s basic fact plan and consider how a shift toward incorporating these ideas could affect the learning and experiences of your students. As you identify a focus and a plan, we hope this book will provide you with the activities, games, and tools to support your own basic fact teaching and assessment transformation.

**FIGURE P.1 Games in This Book**

<table>
<thead>
<tr>
<th>Game</th>
<th>Chapter</th>
<th>Targeted Facts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Game 1: Sleeping Bears</td>
<td>2</td>
<td>Sums within 5</td>
</tr>
<tr>
<td>Game 2: Bears Race to 10</td>
<td>2</td>
<td>+0, 1, 2</td>
</tr>
<tr>
<td>Game 3: Bears Race to 0</td>
<td>2</td>
<td>−0, 1, 2</td>
</tr>
<tr>
<td>Game 4: Bears Race to Escape</td>
<td>2</td>
<td>+/− 0, 1, 2</td>
</tr>
<tr>
<td>Game 5: Doubles Match-Up</td>
<td>2</td>
<td>Doubles (sums)</td>
</tr>
<tr>
<td>Game 6: Doubles Bingo</td>
<td>2</td>
<td>Doubles (sums)</td>
</tr>
<tr>
<td>Game 7: 10 Sleeping Bears</td>
<td>2</td>
<td>Combinations of 10</td>
</tr>
<tr>
<td>Game 8: Go Fish for 10s</td>
<td>2</td>
<td>Combinations of 10</td>
</tr>
<tr>
<td>Game 9: Erase</td>
<td>2</td>
<td>Combinations of 10</td>
</tr>
</tbody>
</table>
### FIGURE P.1  Games in This Book—(continued)

<table>
<thead>
<tr>
<th>Game</th>
<th>Chapter</th>
<th>Targeted Facts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Game 10: Square Deal</td>
<td>2</td>
<td>10 + __ facts</td>
</tr>
<tr>
<td>Game 11: Lucky 13</td>
<td>3</td>
<td>Sums within 20 (and differences from 13)</td>
</tr>
<tr>
<td>Game 12: Sum War</td>
<td>3</td>
<td>Sums within 20</td>
</tr>
<tr>
<td>Game 13: Bingo</td>
<td>3</td>
<td>Sums within 20</td>
</tr>
<tr>
<td>Game 14: Concentration</td>
<td>3</td>
<td>Sums within 20</td>
</tr>
<tr>
<td>Game 15: Dominoes</td>
<td>3</td>
<td>Sums within 20</td>
</tr>
<tr>
<td>Game 16: Four in a Row</td>
<td>3</td>
<td>Sums within 20</td>
</tr>
<tr>
<td>Game 17: Old Mascot (Old Maid)</td>
<td>3</td>
<td>Sums within 20</td>
</tr>
<tr>
<td>Game 18: Diffy Dozen</td>
<td>3</td>
<td>Differences within 12 (comparison)</td>
</tr>
<tr>
<td>Game 19: Salute</td>
<td>3</td>
<td>Sums and differences within 20</td>
</tr>
<tr>
<td>Game 20: Target Difference</td>
<td>3</td>
<td>Differences within 20</td>
</tr>
<tr>
<td>Game 21: Subtraction Stacks</td>
<td>3</td>
<td>Differences of 5 or less</td>
</tr>
<tr>
<td>Game 22: Around the House</td>
<td>3</td>
<td>Sums and differences that equal 10 or less</td>
</tr>
<tr>
<td>Game 23: Dirty Dozen</td>
<td>3</td>
<td>Sums and differences within 12</td>
</tr>
<tr>
<td>Game 24: First to 20</td>
<td>3</td>
<td>Sums and differences within 20</td>
</tr>
<tr>
<td>Game 25: Sticker Book Patterns</td>
<td>4</td>
<td>Comparing multiplication representations (groups and arrays)</td>
</tr>
<tr>
<td>Game</td>
<td>Chapter</td>
<td>Targeted Facts</td>
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<tr>
<td>-------------------------------</td>
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</tr>
<tr>
<td>Game 26: On the Double</td>
<td>4</td>
<td>2s (doubles) facts</td>
</tr>
<tr>
<td>Game 27: Trios</td>
<td>4</td>
<td>5s facts</td>
</tr>
<tr>
<td>Game 28: Capture 5 First</td>
<td>4</td>
<td>2s, 5s, 10s facts</td>
</tr>
<tr>
<td>Game 29: How Low Can You Go?</td>
<td>4</td>
<td>0s, 1s facts</td>
</tr>
<tr>
<td>Game 30: Squares Bingo</td>
<td>4</td>
<td>Squares</td>
</tr>
<tr>
<td>Game 31: Multiplication</td>
<td>4</td>
<td>Foundational multiplication facts</td>
</tr>
<tr>
<td>Pathways</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Game 32: Fixed Factor War</td>
<td>5</td>
<td>Doubling</td>
</tr>
<tr>
<td>Game 33: Strive to Derive</td>
<td>5</td>
<td>Adding/subtracting a group</td>
</tr>
<tr>
<td>Game 34: Crossed Wires</td>
<td>5</td>
<td>Break apart</td>
</tr>
<tr>
<td>Game 35: Rectangle Fit</td>
<td>5</td>
<td>Break apart, commutativity</td>
</tr>
<tr>
<td>Game 36: The Factor Game</td>
<td>5</td>
<td>Division (finding factors)</td>
</tr>
<tr>
<td>Game 37: The Right Price</td>
<td>5</td>
<td>Close-by division facts</td>
</tr>
<tr>
<td>Game 38: Multiplication</td>
<td>5</td>
<td>Multiplication and division facts</td>
</tr>
<tr>
<td>Salute</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Game 39: The Product Game</td>
<td>5</td>
<td>Multiplication and division facts</td>
</tr>
<tr>
<td>Game 40: Net Zero</td>
<td>5</td>
<td>All four operations</td>
</tr>
<tr>
<td>Game 41: Softball Hits</td>
<td>5</td>
<td>All four operations</td>
</tr>
<tr>
<td>Game 42: Three Dice Take</td>
<td>5</td>
<td>All four operations</td>
</tr>
</tbody>
</table>
### FIGURE P.2  Assessment Tools in This Book

<table>
<thead>
<tr>
<th>Title</th>
<th>Chapter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tool 1: Observation Tools for Foundational Fact Sets</td>
<td>6</td>
</tr>
<tr>
<td>Tool 2: Observation Tool for +/– 0, 1, 2</td>
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</tr>
<tr>
<td>Tool 3: Observation Tool for × 2s, 10s, and 5s</td>
<td>6</td>
</tr>
<tr>
<td>Tool 4: Observation Tools for Combinations of 10 and Doubles</td>
<td>6</td>
</tr>
<tr>
<td>Tool 5: Observation Tool for × 5s Facts</td>
<td>6</td>
</tr>
<tr>
<td>Tool 6: Two-Prompt Interview Protocol</td>
<td>6</td>
</tr>
<tr>
<td>Tool 7: Interview Record for Combinations of 10</td>
<td>6</td>
</tr>
<tr>
<td>Tool 8: Interview Record for Multiplication Squares</td>
<td>6</td>
</tr>
<tr>
<td>Tool 9: Mastery of Foundational Facts Records</td>
<td>6</td>
</tr>
<tr>
<td>Tool 10: Rubrics for Foundational Fact Fluency</td>
<td>6</td>
</tr>
<tr>
<td>Tool 11: Journal Writing Prompts for Doubles</td>
<td>6</td>
</tr>
<tr>
<td>Tool 12: Foundational Facts Progress Chart for Multiplication</td>
<td>6</td>
</tr>
<tr>
<td>Tool 13: Observation Tool for the Making 10 Strategy</td>
<td>7</td>
</tr>
<tr>
<td>Tool 14: Observation Tool for Any Multiplication Derived Fact Strategy</td>
<td>7</td>
</tr>
<tr>
<td>Tool 15: Observation Tools for Selection of Strategies</td>
<td>7</td>
</tr>
<tr>
<td>Tool 16: Observation Tool for Strategies and Mastery for Addition Facts</td>
<td>7</td>
</tr>
</tbody>
</table>
Most, if not all, of the 42 games and 26 assessment tools can be readily adapted to other fact sets and operations (e.g., addition games can be turned from sums into products). So, there are truly more than 100 possible games and tools for you—enough to ensure your students are able to truly develop math fact fluency!
The Five Fundamentals

You've probably heard it a thousand times: Kids need to be fluent with basic math facts. You've probably seen the word fluency on progress reports, in elementary mathematics standards, and in textbooks, but what does fluency actually mean? For nearly a decade, we've been asking this question to teachers and administrators. Common responses include

- "They just know the facts."
- "They are fast and accurate."
- "They understand what the fact means."
- "They have strategies to figure out the facts."
- "It's like when you are fluent in a language—you don't have to think or hesitate much."
- "They are automatic with the facts."
- "They can apply their understanding of the facts to new situations."

As you can see, the school community has struggled to embrace a common and comprehensive definition of fluency. Some definitions focus on speed, while others focus on understanding. Reaching the goal of basic fact fluency requires establishing a shared and complete understanding of the term. As baseball great Yogi Berra once noted, "If you don't know where you're going, you'll end up someplace else." This is the tenet behind the first of our five basic fact fundamentals; the four that follow lay out
Math Fact Fluency

essential elements for designing an effective plan—one that will help every student learn (and remember) the basic facts while building mathematical confidence and number sense.

**Fundamental 1: Mastery Must Focus on Fluency**

Procedural fluency includes accuracy, efficiency, flexibility, and appropriate strategy selection (National Research Council, 2001). Note that this definition of procedural fluency applies to all operations, not just basic facts, and these elements of fluency are interrelated (Bay-Williams & Stokes Levine, 2017) as illustrated by the diagram in Figure 1.1.

![Figure 1.1](ADVANCE UNCORRECTED COPY - NOT FOR DISTRIBUTION)

**FIGURE 1.1 What Procedural Fluency Is and What It Looks Like**

- **Procedural Fluency**
  - **Computing with…**
    - **Accuracy**
      - Correct answer
    - **Efficiency**
      - Time needed to solve is reasonable.
    - **Flexibility**
      - Selected strategy “fits” the numbers in the problem (i.e., **Appropriate Strategy Selection**).
      - Strategy is applied to new problems.
      - Strategy is adapted to better fit the problem.

The four components (bolded) are interrelated. Appropriate strategy selection is required for efficiency and flexibility.

Applying strategies is different than applying algorithms. According to the Council of Chief State School Officers (CCSSO) and National Governors Association (NGA), computation strategies are “purposeful manipulations that may be chosen for specific problems,” while algorithms are a “set of predefined steps applicable to a class
of problems” (CCSSO & NGA, 2010, p. 85). When students only learn a single procedure, regardless of how quickly and accurately they can implement it, they are denied the opportunity to develop procedural fluency. Strategy selection, adaptation, and transference are critical to both procedural fluency and mathematical proficiency and must be a significant part of students’ experiences with the operations right from the beginning, with learning basic facts.

We use these general definitions of each component to focus specifically on basic fact fluency:

• **Accuracy**: the ability to produce mathematically precise answers
• **Efficiency**: the ability to produce answers relatively quickly and easily
• **Appropriate strategy use**: the ability to select and apply a strategy that is appropriate for solving the given problem efficiently
• **Flexibility**: the ability to think about a problem in more than one way and to adapt or adjust thinking if necessary

Consider these aspects of fluency in terms of level of cognition. Which of these requires higher-level thinking? Selecting a strategy, key to both efficiency and flexibility, requires first understanding how and when each strategy is appropriate, and then analyzing a problem to select a viable strategy. Notice that fluency requires understanding, applying, analyzing, and comparing—all higher-level thinking processes. The more students are asked to think at a higher level, the more they learn.

Basic facts are also described in terms of fluency in state and national standards, such as in these examples from the Common Core State Standards (CCSSO & NGA, 2010) (emphasis added):

1.OA.6 (Grade 1): Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). (p. 15)

2.OA.2 (Grade 2): Fluently add and subtract within 20 using mental strategies [with a reference to 1.OA.C.6]. By end of Grade 2, know from memory all sums of two one-digit numbers. (p. 19)

3.OA.7 (Grade 3): Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing
that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. (p. 23)

These standards acknowledge that it is through the application of strategies that a student develops fluency, and it is through the use of strategies that students come to know their basic facts, or develop automaticity (more on this point in the next section). However, the activities and assessments traditionally associated with learning basic facts (such as drill, flash cards, and timed testing) exclusively focus on students’ accuracy and one part of efficiency (speed), neglecting strategy development. Many studies over many years have compared traditional basic fact instruction (i.e., drill) to strategy-focused instruction. All of them show that strategy groups outperform their peers on using strategies and on automaticity and accuracy (Baroody, Purpura, Eiland, Reid, & Paliwal, 2016; Brendefur, Strother, Thiede, & Appleton, 2015; Locuniak & Jordan, 2008; Purpura, Baroody, Eiland, & Reid, 2016; Thornton, 1978, 1990; Tournaki, 2003). We know that strategy development is absolutely necessary for fluency. And fluency is essential to developing automaticity with basic facts.

**Fundamental 2: Fluency Develops in Three Phases**

As students come to know basic facts in any operation, they progress through three phases (Baroody, 2006):

- **Phase 1: Counting** (counts with objects or mentally)
- **Phase 2: Deriving** (uses reasoning strategies based on known facts)
- **Phase 3: Mastery** (efficiently produces answers)

Consider these phases in the context of mastering addition facts. Most students enter kindergarten or 1st grade using counting to solve addition or subtraction problems. They may be counting with objects, on their fingers, or in their heads, but, regardless, these students are still considered to be at Phase 1. As they start to learn some of the easier facts (usually $2 + 2 = 4$, $3 + 3 = 6$, and $5 + 5 = 10$), they can begin using those facts to help them to figure out more difficult, related facts. For example, to find $5 + 7$, a student might begin with $5 + 5 = 10$ and add on two more to determine that $5 + 7 = 12$. This is an example of Phase 2 thinking, where the answer to a more challenging fact is being derived by using a known fact. The flexibility, increased efficiency, and selection of appropriate strategies that are developed in this phase are critical to fluency.
As students engage in sufficient meaningful practice in Phase 2, they become faster in their strategy selection and application and come to know some facts without needing to apply a strategy. Thus, they move naturally into Phase 3 (mastery), which is characterized by the highly efficient production of answers, either through quick strategy application or through recall. Students operating at Phase 3 are considered automatic with those facts, as they meet the definition commonly accepted for automaticity—answering within three seconds, either through recall or automatic strategy application (Van de Walle, Karp, & Bay-Williams, 2019). Thus, the difference between Phase 2 and Phase 3 is essentially speed; in both cases, students may be applying appropriate strategies flexibly, but students at Phase 3 answer instinctively within a few seconds, whereas Phase 2 students might take longer to select and apply a strategy.

You’ve likely seen advertisements for books or fact-learning programs that promise “fact fluency in two minutes a day” or “know your facts in seven days.” The reality is there are no shortcuts to developing fluency or to mastery and automaticity. “Quick fix” programs attempt to take students who are operating at Phase 1 (counting) and push them directly to Phase 3, usually through drill and timed testing, skipping any effort to explicitly teach strategies or focus on number relationships. Students subjected to such programs may appear to know the facts in the short term, but within weeks or months they are back to where they started: counting. Because little to no time is spent in Phase 2, once facts are forgotten, students have no efficient, appropriate, and flexible strategies to fall back on. This explains why we sometimes see middle grade students counting to solve basic facts, much to the chagrin of their teachers. In contrast, to encourage lasting mastery of basic facts, students need to have sufficient time and experiences in Phase 2. The activities, games, and assessment tools you will find in this book are designed to do just that.

**Fundamental 3: Foundational Facts Must Precede Derived Facts**

Perhaps you have memories of learning groups of multiplication facts in order. You memorized the 0s facts, passed a test (and perhaps got a sticker on your chart), and then moved on to the 1s, 2s, 3s, and so on. Although once common, this sequence is not consistent with what research suggests is the most effective approach to learning facts. There are sets of facts within both addition and multiplication that are easier for students to master first and are essential to applying derived fact strategies. We
refer to these facts as foundational fact sets, or foundational facts for short. A foundational fact set is a set of facts that illustrate a specific pattern or number relationship. For example, working on the one less facts can be connected to the counting sequence (the number that comes before), to the number line (the number that is one to the left), and to the idea of taking away one.

The remaining facts can be derived from the foundational facts through strategy application. Thus, these sets of facts are called derived facts, and students come to know these facts by learning derived fact strategies. Notice that we do not use the term subset with derived facts. That is because many derived facts can be reasonably solved using more than one derived fact strategy. In fact, students must have many opportunities to select which of the derived fact strategies they will use to solve a combination that they do not know. A flexible learning progression demonstrating the relationships between facts for addition is presented in Figure 1.2. Many studies have found that mathematics teaching based on learning progressions leads to positive effects on children's early math achievement (Frye et al., 2013).

In this chart we include the +/- 0, 1, and 2 facts as foundational and the place to begin. Notice that each of the other foundational facts, except perhaps doubles, can flow easily from already knowing +/- 0, 1, and 2. Therefore, to work toward mastery of
all facts, a first step is to develop automaticity with the \(+/- 0, 1,\) and \(2\) facts. At the next level are more foundational facts, which can be taught in a flexible order, as mastery of one is not needed to reach mastery of another. However, students must master specified foundational facts to use the related derived fact strategies on the final level (e.g., Doubles must precede Near Doubles).

Similarly, multiplication facts can be taught in groupings so that known foundational facts can be used to derive other facts. The flexible learning progression for multiplication is shown in Figure 1.3.

![Figure 1.3: Multiplication Fact Fluency Flexible Learning Progression](image)

The distinction between foundational fact sets and derived fact strategies is essential for effective teaching of the basic facts because it provides a blueprint for monitoring fact instruction progress. Consider the example of finding \(5 \times 7\). If a student didn’t already know that \(5 + 5 = 10\), he would not be able to arrive at the solution as described: \(5 + 5 = 10, 10 + 2 = 12,\) so \(5 + 7 = 12\). Thus, when we observe students who are unable to use a strategy for finding this fact, we must determine if they have learned the foundational facts to automaticity. If not, that is where intervention must be focused.
Because this progression is so important to students’ success with using strategies to master basic facts, we have organized this book around the two groupings of foundational facts and derived fact strategies. Chapters 2 and 4 focus on teaching the foundational fact sets for addition and multiplication, respectively, and Chapters 3 and 5 do the same for derived fact strategies. Chapters 6 and 7 focus on assessing foundational fact sets and derived fact strategies, respectively. The two flexible learning progressions will appear throughout the book to highlight the fact sets or strategies discussed. Our hope is that these charts will not only help you visualize the progression of fact mastery in a typical classroom but also help you with individual progress monitoring in order to develop plans of action to support students who have not yet mastered all the basic facts.

**Fundamental 4: Timed Tests Do Not Assess Fluency**

Picture a worksheet containing 100 multiplication facts in random order, which students are asked to complete in five minutes. Perhaps you remember these timed tests from your childhood, or perhaps you still see these in use in classrooms today. Now determine how many of the four components of fluency (flexibility, accuracy, efficiency, and appropriate strategy use) you believe are actually assessed with a timed test. We've posed this task countless times to many groups of teachers and administrators, most of whom have initially thought that, at most, two components are assessed—but which two? Flexibility and appropriate strategy use are easily eliminated. Because the teacher only sees a recorded answer, it is impossible to assess if a student is flexible or chooses appropriate strategies from a timed test alone. This does not mean that students aren’t flexible or that they don’t use appropriate strategies; the timed test simply doesn’t allow a teacher to see it. What about efficiency and accuracy? Although at first glance it may seem that a timed test can assess these components, there are certainly instances where this isn’t true. Consider the following scenarios.

Tommy is taking his weekly multiplication test. Although he has learned many easier multiplication facts, Tommy still struggles to remember his 7s, and he is very aware of this weakness. Once again, he compensates by skipping around and answering the facts he knows; then he quietly puts his hands under his desk to help him count to answer the remaining, unknown facts. He has learned that he can usually finish the test in time by doing this, and his teacher is therefore convinced he knows his facts.
Ellie feels her heart start to race when her teacher announces it is time to start the weekly addition facts timed test. Even though Ellie excels at reading, writing, and solving even the most challenging story problems, as soon as the timer starts, she draws a blank. She struggles to remember the facts she knows well and is so distracted by the timer that she can't apply her favorite strategies to tougher addition facts, like $7 + 8$. With tears in her eyes, she once again turns in an incomplete test and tells her friends she's "just so bad at math." Her teacher is puzzled; she has seen Ellie's automaticity with addition facts many times during math games and doesn't understand why that doesn't translate to the test.

Whether it be from our own childhood experiences or from experiences as an adult, we've all known students like Tommy and Ellie. Let's look at Tommy's fluency. Even though he knew some of the facts, the completed, correct answers on his timed test give the illusion of mastery. The reality is that he is not efficient with all the facts (namely, the 7s), and yet his ability to "play the game" has not only fooled his teacher but also reinforced that he doesn't need to make an effort to learn those facts. Tommy's case illustrates how timed testing does not provide a wide enough lens for evaluating fluency, because it doesn't reveal the exact facts with which students are efficient.

Next, consider Ellie's fluency. She is an excellent mathematical thinker, loves to write and solve problems, and, based on her teacher's observations during game play, has mastered the addition facts. Yet the pressure of time cripples her thinking and essentially invalidates her test as a measure of accuracy. Even worse, this experience has convinced Ellie that she is bad at math when, in fact, she is quite the opposite! Ellie is not unique. In fact, over the past decade there have been numerous findings from psychology and even neuroscience uncovering the damaging effects of timed testing. For example, in a study of more than 50 students in 1st and 2nd grades, Ramirez, Gunderson, Levine, and Beilock (2013) found that students begin experiencing math anxiety as early as 1st grade and that anxiety was not correlated with reading achievement or socioeconomic status. However, they did find an important, troubling correlation: The students who tended to use more sophisticated mathematical strategies were those who often experienced the most negative impact on achievement due to math anxiety. In other words, by age 7, many young students with high mathematical aptitudes are already learning to fear math. Boaler (2012, 2014) also reported that even students who perform well on timed tests share concerns such as "I feel nervous" and "I know my facts, but this just scares me."

Timed testing is often considered synonymous with learning basic facts, and yet, as we have just described, it is highly ineffective at assessing any of the four
components of fluency. Why, then, is it still so common? Some schools feel that timed testing is necessary for promoting fact mastery. However, there is no evidence to support this theory. In fact, there is evidence to the contrary. In a study of nearly 300 1st graders, Henry and Brown (2008) found that those students who were more frequently exposed to timed testing actually demonstrated slower progress toward automaticity with their facts than their counterparts who were not tested.

Why are timed tests still so prevalent given the evidence that they don’t work? We think many schools continue to use timed testing because they simply do not know how else to assess fact mastery. We hope to rectify this issue and offer a variety of formative assessments in Chapters 6 and 7. These assessment tools and techniques allow teachers to assess all four components of fluency while still encouraging mathematics confidence in their students.

**Fundamental 5: Students Need Substantial and Enjoyable Practice**

Substantial and enjoyable practice should be considered as an alternative to timed drills for developing mastery with basic facts. Imagine, for example, posing this question to 1st or 2nd grade students: *How many equations can you write that equal 10?* Students enjoy open-ended challenges, and the task allows for natural differentiation, with some students inventing equations with three and four addends and others simply listing known facts. In looking at the ways the students thought about their equations, you learn what number relationships they know—and, as you will soon read, knowing how far numbers are from 10 is an essential concept.

These same students may also play Go Fish for 10s, a game where a match is a combination of 10 (a student with a 4 asks, “Do you have any 6s?”). As you will see later in this book, many familiar games can be adapted to practice basic facts, including Four in a Row, Concentration, and War. We will also share many novel basic fact games, such as Crossed Wires, in which students create grids (arrays) of crossed wires to practice derived fact strategies for multiplication.

Games and other enjoyable challenges provide ample fact practice without constantly using those pages with 100+ facts. Additionally, games are interactive, so students can think aloud and hear others’ strategies (Bay-Williams & Kling, 2014; Godfrey & Stone, 2013). Think-aloud opportunities are beneficial to all students but are particularly effective with students who traditionally struggle to learn mathematics (Frye et al., 2013; Gersten & Clarke, 2007).
Furthermore, during game play, the teacher has an opportunity to implement formative assessment tools to monitor each student’s progress toward mastery. However, it is not enough for a game to be fun; it also has to provide a meaningful mathematics experience. The features described below provide guidance on how to select games that will, in fact, provide effective fact practice. Although a game may not reflect all these features, any game that has most of these features will more effectively help students’ fluency development.

### 10 Questions to Guide Game Selection

To what extent does the game . . .

1. Provide an opportunity to practice the subset of facts that the students are learning?
2. Appeal to the age of your students?
3. Employ visuals or tools (such as ten frames, quick looks, or arrays) to support strategy development?
4. Involve selecting from among derived fact strategies (for mastery-level games)?
5. Provide opportunities for discussion among students about their mathematical thinking?
6. Encourage individual accountability? (For example, are students solving their own facts or competing to solve the same fact? The former practice provides more “think time” and avoids opting out.)
7. Remove time pressures?
8. Involve logic or strategic moves, enhancing the “fun factor”?
9. Offer opportunities for adaptation so that all students can experience appropriate challenge?
10. Lend itself to you being able to listen and watch in order to assess progress?

Sometimes you need a game that is focused on one set of foundational facts (e.g., 5s facts for multiplication); at other times, you need a game that requires that each student identify a derived fact strategy (e.g., deciding how to break apart one factor to use known facts). When working on derived facts, students need think time. Therefore, when selecting a game to help students practice derived fact strategies, a good choice is one in which time is not a factor and where each player is finding a different fact so that students are not trying to find an answer faster than their partner. Once students are automatic with all of their facts, foundational and derived, then games
that involve speed may be appropriate and enjoyable. As students get older, the more strategic a game is, the more fun it is to play. Games like Connect Four, for example, involve trying to both get four in a row yourself and block your partner. In summary, good game selection requires various considerations: age of the student, the facts being learned, and student fluency with that set of facts.

Within the covers of this book are more than 40 games that reflect, at least to some extent, the features of effective games described above. Nearly all of these games are readily adaptable to other fact sets or operations, resulting in well over 100 versions that provide enjoyable, targeted, strategy-focused ways to move students toward Phase 3 (mastery). Additionally, many other games exist online and in various books. The features above can be used to evaluate the quality of these games in supporting students’ emerging fluency and automaticity.

Let’s Get Started!

Hopefully this chapter has piqued your interest for the need to change how fact fluency is developed and assessed. We have a lot of work ahead! By the time you have finished reading this book, you will

- Develop an understanding of foundational facts for each operation.
- Develop an understanding of research-based derived fact strategies for each operation.
- Learn how to sequence facts instruction to best promote natural strategy development and eventual fluency and automaticity.
- Explore activities and games for helping students progress through the phases of fact mastery.
- Consider a variety of assessment tools that can monitor fact mastery in informative and supportive ways.

Let’s get started!
References


Jennifer M. Bay-Williams, PhD, is a mathematics teacher educator at the University of Louisville, Kentucky. She has written many articles and books related to K–12 mathematics education, including the popular Elementary and Middle School Mathematics: Teaching Developmentally and the related three-book series, Teaching Student-Centered Mathematics. Other recent books include Everything You Need for Mathematics Coaching, On the Money (financial literacy), and Developing Essential Understanding of Addition and Subtraction. Bay-Williams is a national leader in mathematics education, having served as a member of the National Council of Teachers of Mathematics (NCTM) Board of Directors, secretary and president of the Association of Mathematics Teacher Educators (AMTE), lead writer for the Standards for Preparing Teachers of Mathematics (AMTE, 2017), and a member of the TODOS: Mathematics for ALL Board of Directors. Bay-Williams taught elementary, middle, and high school students in Missouri and in Peru. She currently works in elementary classrooms in the Louisville area, helping teachers and students attain basic fact fluency while also developing strong mathematical identities. Follow Bay-Williams on Twitter (@JBayWilliams) or contact her directly at j.baywilliams@louisville.edu.

Gina Kling is fortunate to serve the mathematics education community in a variety of ways. Since 2011, she has worked as a curriculum developer for the elementary mathematics curriculum Everyday Mathematics (based at the University of Chicago) with a focus on grades K–3. Recently she served as the grade 1 lead author for the Everyday Mathematics 4 State Editions, the author of the Everyday Mathematics 4...
Quick Looks Activity Book, and one of the authors of Everyday Mathematics for Parents: What You Need to Know to Help Your Child Succeed. Kling has taught mathematics content and methods courses for the past 15 years at Western Michigan University in Kalamazoo, Michigan, and is also currently completing a doctoral degree in K–12 mathematics education at Western Michigan University. For more than a decade, Kling has focused her research on helping children learn basic math facts in meaningful ways and often shares her work through professional development with practicing teachers across the country. She has authored numerous articles on teaching and assessing basic facts and remains active in the elementary classroom today as a mathematics coach, engaging children in developing fact fluency. Kling can be contacted directly at gina.garza-kling@wmich.edu.
Related ASCD Resources

At the time of publication, the following resources were available (ASCD stock numbers in parentheses).

**Books**

*Building a Math-Positive Culture: How to Support Great Math Teaching in Your School (ASCD Arias)* by Cathy L. Seeley (#SF116068)

*Lesson Imaging in Math and Science: Anticipating Student Ideas and Questions for Deeper STEM Learning* by Michelle Stephan, David Pugalee, Julie Cline, and Chris Cline (#117008)

*Making Sense of Math: How to Help Every Student Become a Mathematical Thinker and Problem Solver (ASCD Arias)* by Cathy L. Seeley (#SF116067)

*The School Leader’s Guide to Building and Sustaining Math Success* by Marian Small and Doug Duff (#118039)

*Teaching Students to Communicate Mathematically* by Laney Sammons (#118005)

*Unpacking Fractions: Classroom-Tested Strategies to Build Students’ Mathematical Understanding* by Monica Neagoy (#115071)

**Quick Reference Guides**


*Games and Tools for Teaching Multiplication Facts* (Quick Reference Guide) by Gina Kling and Jennifer Bay-Williams (#QRG119016)

*Guiding Meaningful Math Conversations* (Quick Reference Guide) by Laney Sammons (#QRG117056)

For up-to-date information about ASCD resources, go to [www.ascd.org](http://www.ascd.org). You can search the complete archives of *Educational Leadership* at [www.ascd.org/el](http://www.ascd.org/el).

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